# Lab-2: Converting a physical distance to a grid distance using least-square method

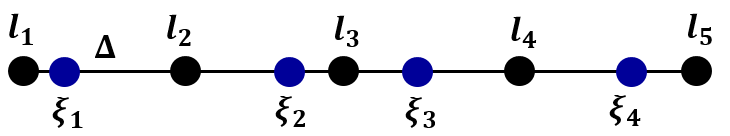
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## Objective:

The objective of this laboratory work is to formalize a problem of placing discrete measurements on a grid scale to cover all the area of interest. This is needed for processing space data to study multi-dimensional processes. The important outcome of this exercise is the solution of problem without using classical regression models and thus avoiding bulky computations and creation of regression matrix of high dimension.

## Problem:

Develop a grid scale with length of bin equal to by minimizing distance between measurements and grid nodes . That means to find starting grid node and bin length . The number of grid nodes is equal to .



## Solution:

Let’s denote grid nodes. – distance between grid nodes.

Satellite measurements are available at sequent times .

But distance between measurements is variable

*,*  – random uncorrelated unbiased noise with constant variance

1. Let’s introduce a vector with two components and that should be found
2. Let’s introduce a matrix-row
3. Find a derivative of functional with respect to

The derivative of scalar multiple of vectors in respect to vector

And minimize the functional the derivative should be equal to zero

1. Let’s introduce the following denotations

Where

Then

Sum of natural numbers:

Then

n = length(KSI);  
  
W = [ n (n-1)\*n/2 ;  
 (n-1)\*n/2 (n-1)\*n\*(2\*n-1)/6 ];

1. Let’s introduce the following denotations

C = [ sum(KSI);  
 sum(KSI.\*((1:n) - 1)) ];

1. Determine unknown vector from Equation (7)

Let’s multiply both parts on

Then

X = W \ C;

## Determine the covariance matrix of estimation error of vector

L = X(1)+(0:n-1)\*X(2);  
varX = sum((KSI - L).^2) / (n - 2);  
covX = varX .\* inv(W);

## Appendix 1: MatLab Source code:

%% Lab-2: Converting a physical distance to a grid distance using least-square method  
% Team1: Dmitry Shadrin and Eugenii Israelit, Skoltech, 01.04.2016 v1  
  
%% Prepare Data  
clc; clear; close all;  
KSI = importdata('ksi2\_N\_100\_sigma\_7\_L1\_1000\_delta\_10.mat');  
% KSI = importdata('ksi1\_N\_100\_000\_sigma\_7\_L1\_1000\_delta\_10.mat');  
  
%% Determine unknown vector X  
n = length(KSI);  
  
W = [ n (n-1)\*n/2 ;  
 (n-1)\*n/2 (n-1)\*n\*(2\*n-1)/6 ];  
  
C = [ sum(KSI);  
 sum(KSI.\*((1:n) - 1)) ];  
  
X = W \ C;  
  
%% Determine the covariance matrix of estimation error of vector X  
L = X(1)+(0:n-1)\*X(2);  
varX = sum((KSI - L).^2) / (n - 2);  
covX = varX .\* inv(W);